Distributed Optimization for Machine Learning

Lecture 13 - Distributed Training: A Close Look at Data Parallelism

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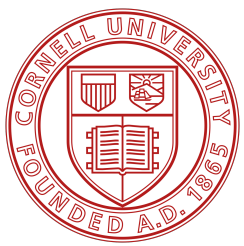
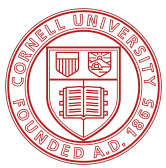
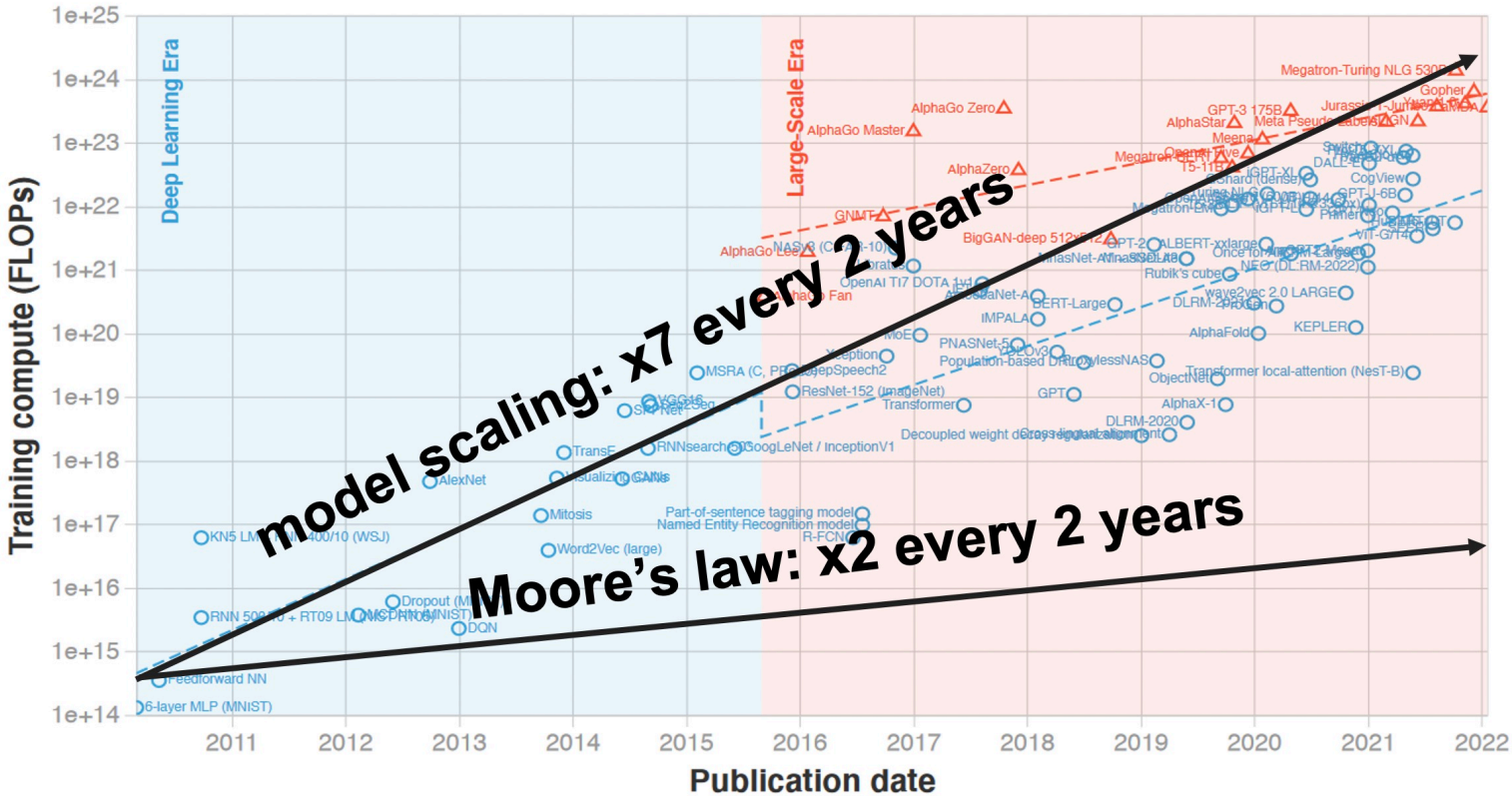


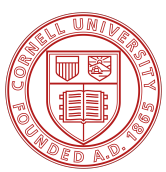
Table of Contents



Training large models requires large compute



Sevilla et al., "Compute trends across three eras of machine learning," **IJCNN 2022.**

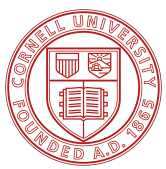


Why parallelize and distributed training?

Modern LLMs push **parameter count**, **context length**, and **training tokens** to extremes.

- **Memory**: weights + optimizer states + activations.
- **Throughput**: tokens/sec limited by compute and communication.

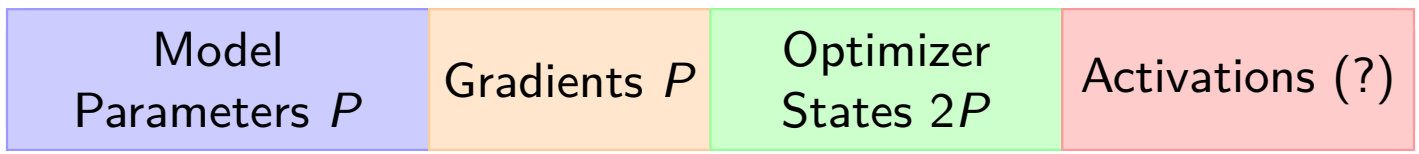
Larger scale + larger memory



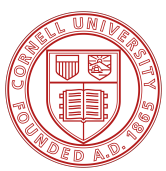
Memory decomposition of training LLMs

Back-of-the-envelope memory (dense FP16 training):

$$\textbf{Memory} = \underbrace{\text{Model}}_{\text{weights}} + \underbrace{\text{Gradients}}_{\text{backprop}} + \underbrace{\text{Optimizer States}}_{\text{e.g., Adam m,v}} + \underbrace{\text{Activations}}_{\text{forward/backward pass}}$$



Typical memory usage breakdown in training large models.



Memory comparison between GPT-3 vs LLaMA

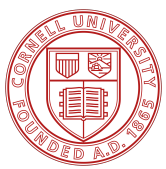
GPT-3 175B

- Weights (FP16): $175 \times 10^9 \cdot 2 \approx$ **350 GB**
- *Training state* ($\sim$16 B/param): $\approx$ **2.8 TB**
- Token storage (e.g., 300B tokens @2 B/token): $\sim$ *600 GB* raw text

LLaMA 65B

- Weights (FP16): $65 \times 10^9 \cdot 2 \approx$ **130 GB**
- *Training state* ($\sim$16 B/param): $\approx$ **1.04 TB**
- If trained on $\sim$1T tokens @2 B/token: $\sim$ *2 TB* raw text

Implication: A *single GPU* cannot hold full training state for 65B-175B dense models. We need **parallelism:** data, model (tensor/pipeline), or hybrids (3D).



Thousands of GPUs are needed to train LLMs

2 example models

GPT-3 (2020)

50,257 vocabulary size
2048 context length
175B parameters
Trained on 300B tokens

Model Name	n_{params}	n_{layers}	d_{model}	n_{heads}	d_{head}	Batch Size	Learning Rate
GPT-3 Small	125M	12	768	12	64	0.5M	6.0×10^{-4}
GPT-3 Medium	350M	24	1024	16	64	0.5M	3.0×10^{-4}
GPT-3 Large	760M	24	1536	16	96	0.5M	2.5×10^{-4}
GPT-3 XL	1.3B	24	2048	24	128	1M	2.0×10^{-4}
GPT-3 2.7B	2.7B	32	2560	32	80	1M	1.6×10^{-4}
GPT-3 6.7B	6.7B	32	4096	32	128	2M	1.2×10^{-4}
GPT-3 13B	13.0B	40	5140	40	128	2M	1.0×10^{-4}
GPT-3 175B or "GPT-3"	175.0B	96	12288	96	128	3.2M	0.6×10^{-4}

Table 2.1: Sizes, architectures, and learning hyper-parameters (batch size in tokens and learning rate) of the models which we trained. All models were trained for a total of 300 billion tokens.

Training: (rough order of magnitude to have in mind)

- O(1,000 - 10,000) V100 GPUs
- O(1) month of training
- O(1-10) $M

LLaMA (2023)

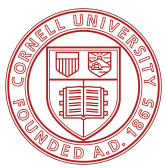
32,000 vocabulary size
2048 context length
65B parameters
Trained on 1-1.4T tokens

params	dimension	n heads	n layers	learning rate	batch size	n tokens
6.7B	4096	32	32	$3.0e^{-4}$	4M	1.0T
13.0B	5120	40	40	$3.0e^{-4}$	4M	1.0T
32.5B	6656	52	60	$1.5e^{-4}$	4M	1.4T
65.2B	8192	64	80	$1.5e^{-4}$	4M	1.4T

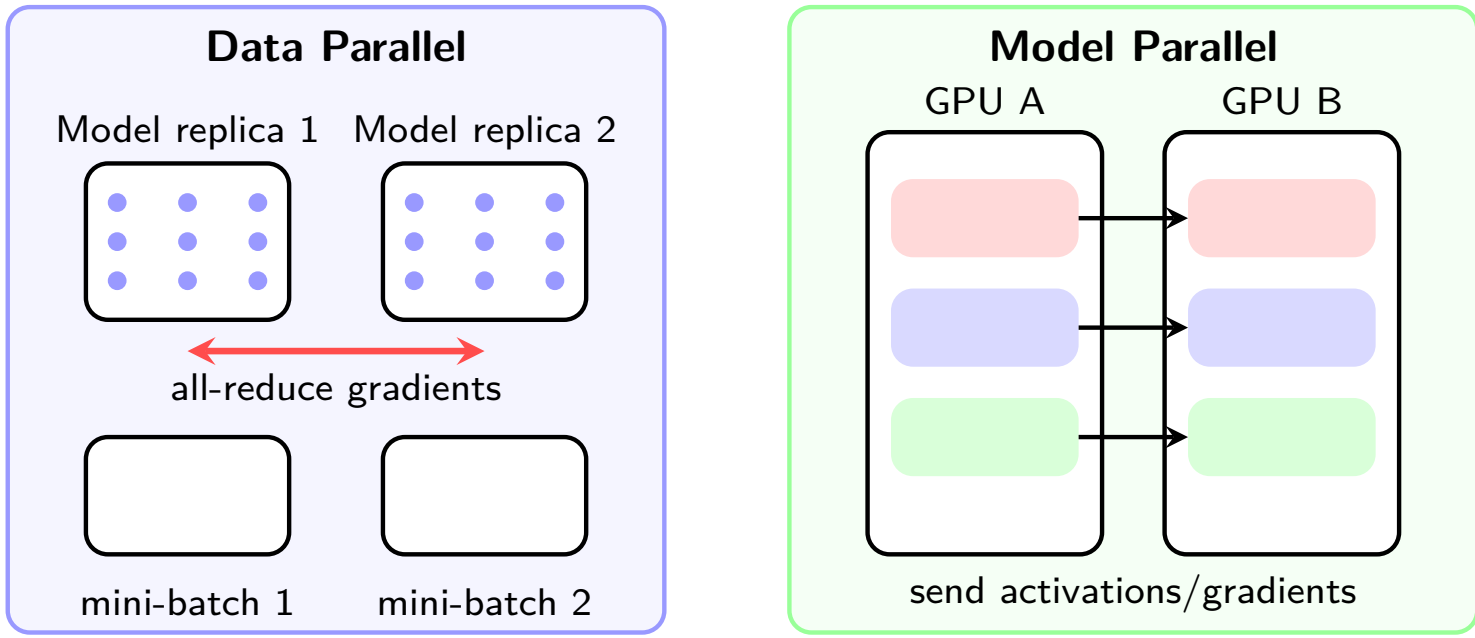
Table 2: **Model sizes, architectures, and optimization hyper-parameters.**

Training for 65B model:

- 2,048 A100 GPUs
- 21 days of training
- $5M

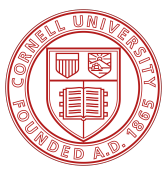


Two paths: Data parallel vs. model parallel



Data parallel: replicate the model; split the data; *consensus* at each step.

Model parallel: split the model across devices; exchange activations.



Why start with data parallel? Memory arithmetic

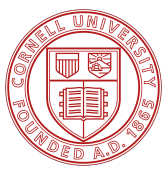
Let P be parameters, b bytes/param for *weights*, s bytes/param for *optimizer & master*, and A activation memory per batch.

$$\text{Per-GPU memory (DP)} \approx \frac{P \cdot b + P \cdot s}{1} + A \quad \text{(replicated across GPUs)}.$$

- If this **fits** in one GPU, add GPUs with DP to increase tokens/sec.
- If it **doesn't fit**, you must shard with model parallel across GPUs:

$$\text{Per-GPU memory (MP)} \approx \frac{P \cdot b + P \cdot s}{n_{\text{shards}}} + \text{per-stage activations}$$

- Combine with activation checkpointing to push the limit.

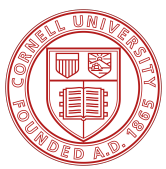


When do we need each?

- **Data Parallel**: model + optimizer *fit* on a single GPU, but you want more throughput
  - Near-linear speedup until communication (all-reduce) dominates.
  - Combine with compression, mixed precision, and large batch training.

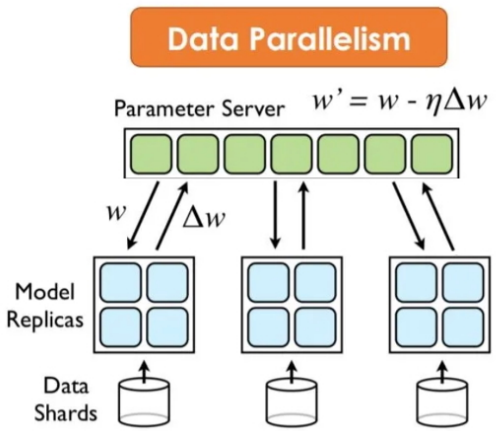
- **Model Parallel**: model *does not fit* (weights/activations/optimizer)
  - *Tensor parallel*: shard matrices within layers.
  - *Pipeline parallel*: shard layers across stages

Design target for different parallelisms: minimize *communication/compute* ratio while keeping memory under budget.

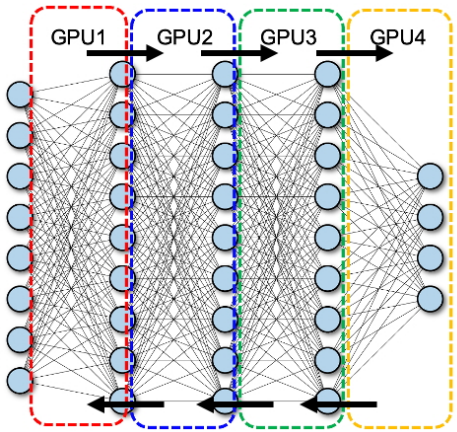


3 ways of parallelism used to train large models

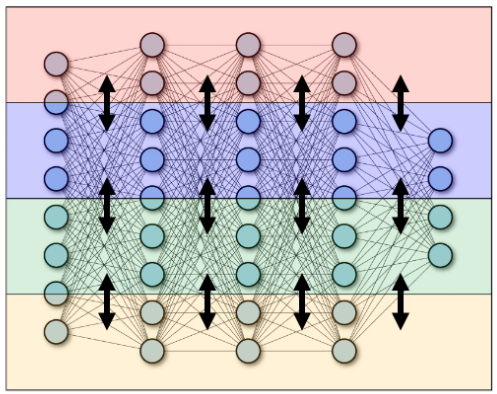
In today's industry, there are three dimensional-parallelism indicates 3 orthogonal parallel techniques used to train large models such as LLMs.



Data parallelism



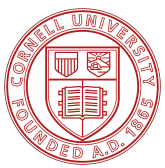
Pipeline parallelism



Tensor parallelism

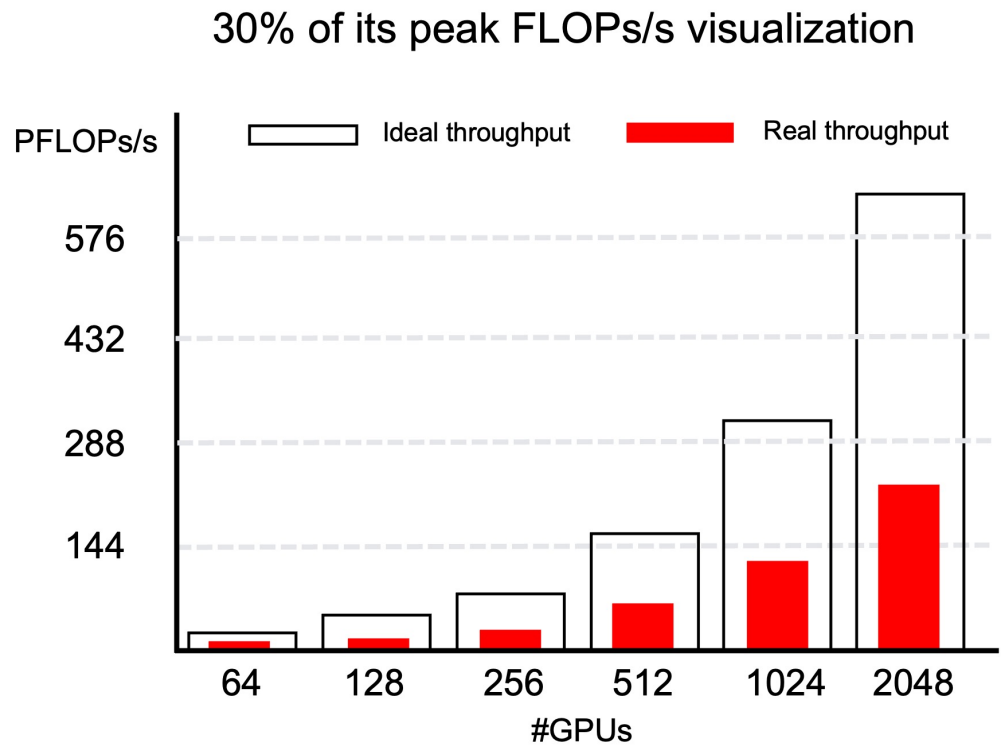
model parallelism

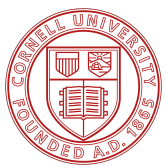
This lecture mainly focuses on **data parallelism**.



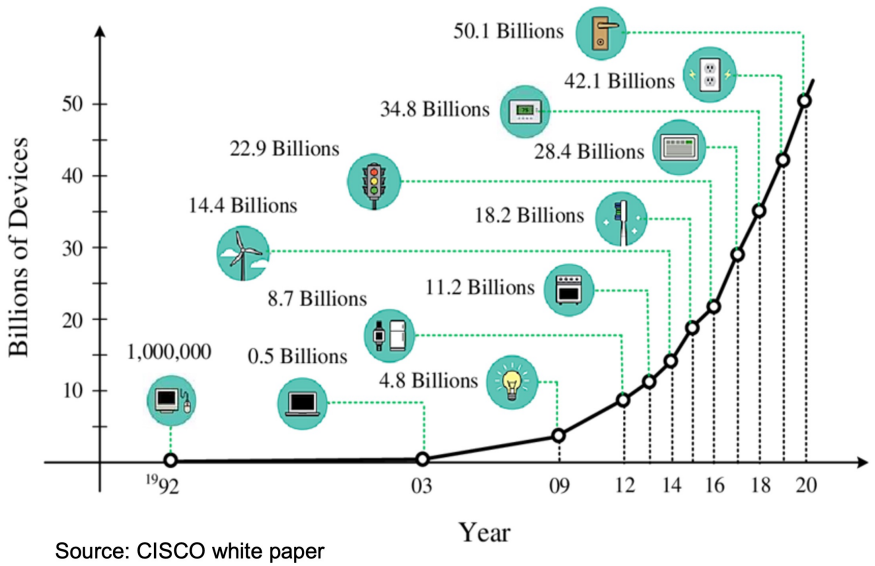
Distributed training is extremely challenging

- The communication overhead and GPU idle time hamper scalability
- Each GPU can only achieve 30%-55% of its peak compute power
- The system achieves 30% scalability - inefficient





Distributed training beyond data centers



Source: CISCO white paper

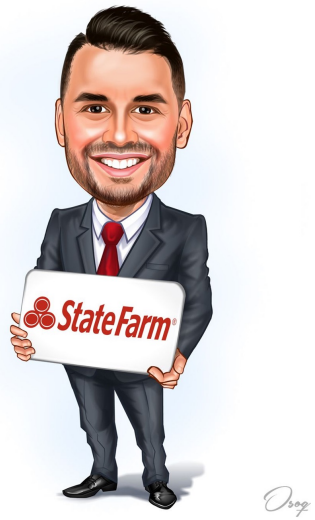
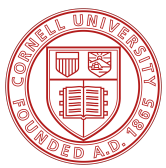


Table of Contents

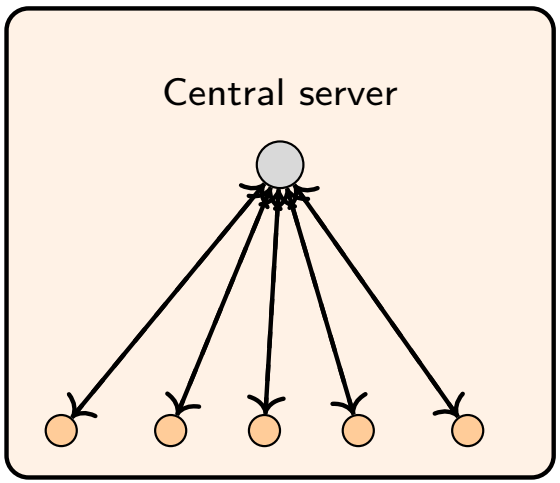


Optimization formulation of data parallelism

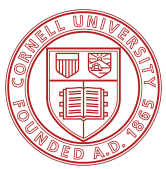
A network of n nodes (here are GPUs) collaborate to solve:

$$\min_{\mathbf{x}\in\mathbb{R}^d} f(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} f_i(\mathbf{x}), \quad \text{where } \boxed{f_i(\mathbf{x}) = \mathbb{E}_{\xi_i\sim D_i}[F(\mathbf{x};\xi_i)]}$$

- Each component $f_i : \mathbb{R}^d \to \mathbb{R}$ is local and private to node i.
- Random variable ξ_i denotes local data following distribution D_i.
- D_i may be different $\Rightarrow$ **data heterogeneity**.



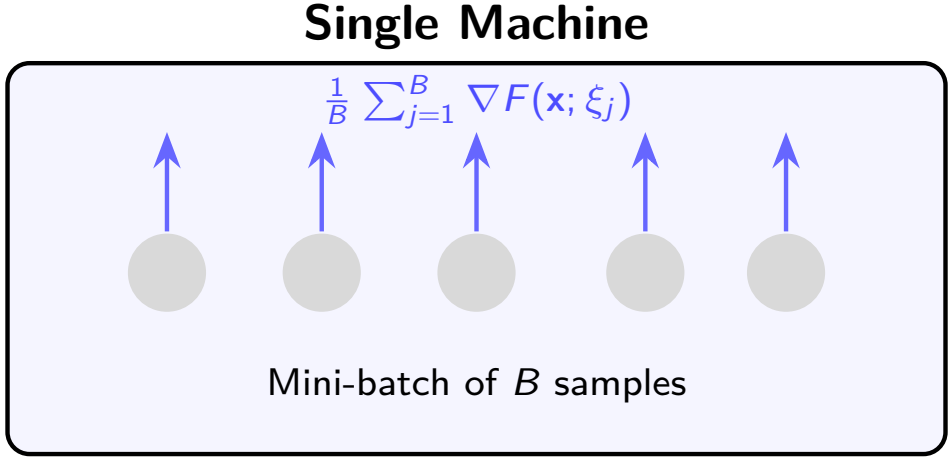
Local data on nodes



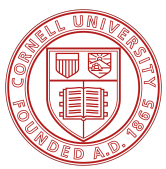
Mini-batch SGD: Averaging within one machine

Key idea: Instead of updating per sample, average gradients over a mini-batch of samples.

$$\mathbf{x}^{k+1} = \mathbf{x}^k - \frac{\eta}{B} \sum_{j=1}^{B} \nabla F(\mathbf{x}^k; \xi_j^k)$$



Effect: Reduces variance and stabilizes convergence.



Parallel SGD: compute locally, communicate globally

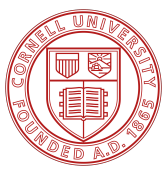
$$\min_{\mathbf{x}\in\mathbb{R}^d} \; f(\mathbf{x}) \;=\; \frac{1}{n}\sum_{i=1}^{n} f_i(\mathbf{x}), \qquad \text{where } f_i(\mathbf{x}) = \mathbb{E}_{\xi_i\sim D_i}[F(\mathbf{x};\xi_i)]\,.$$

PSGD

$$g_i^k \;=\; \nabla F\left(\mathbf{x}^k;\, \xi_i^k\right) \qquad \text{(Local compt.)}$$

$$\mathbf{x}^{k+1} \;=\; \mathbf{x}^k \;-\; \frac{\eta}{n}\sum_{i=1}^{n} g_i^k \qquad \text{(Global comm.)}$$

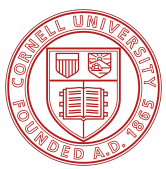
- Each node i samples mini-batch ξ_i^k and computes $\nabla F\left(\mathbf{x}^k;\xi_i^k\right)$.
- All nodes synchronize (i.e., *globally average*) to update $\mathbf{x}$.



Mini-batch vs. Parallel SGD implementation

Aspect	**Mini-Batch SGD**	**Parallel SGD**
Computation	One machine averages over B samples	n workers compute on separate batches
Synchronization	Implicit (within memory)	Explicit (via Allreduce or parameter server)
Effective batch size	B	$n \times B_{\text{local}}$
Noise reduction	By sampling multiple data	By averaging across workers
Communication	None	Required per iteration
Data heterogeneity	N/A (one dataset)	Possible ($D_i \neq D_j$)

Key: Parallel SGD = Mini-batch SGD distributed across multiple nodes.



From consensus to Ring-AllReduce

Recall: In consensus averaging, all workers iteratively mix local states

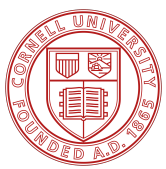
$$\mathbf{x}_i^{k+1} = \sum_j W_{ij}\mathbf{x}_j^k, \quad \text{where } \mathbf{W} \text{ is doubly stochastic.}$$

Goal: Reach *consensus average* $\mathbf{x}_i^k \to \bar{\mathbf{x}} = \frac{1}{n}\sum_{i=1}^n \mathbf{x}_i$.

Connection to distributed training

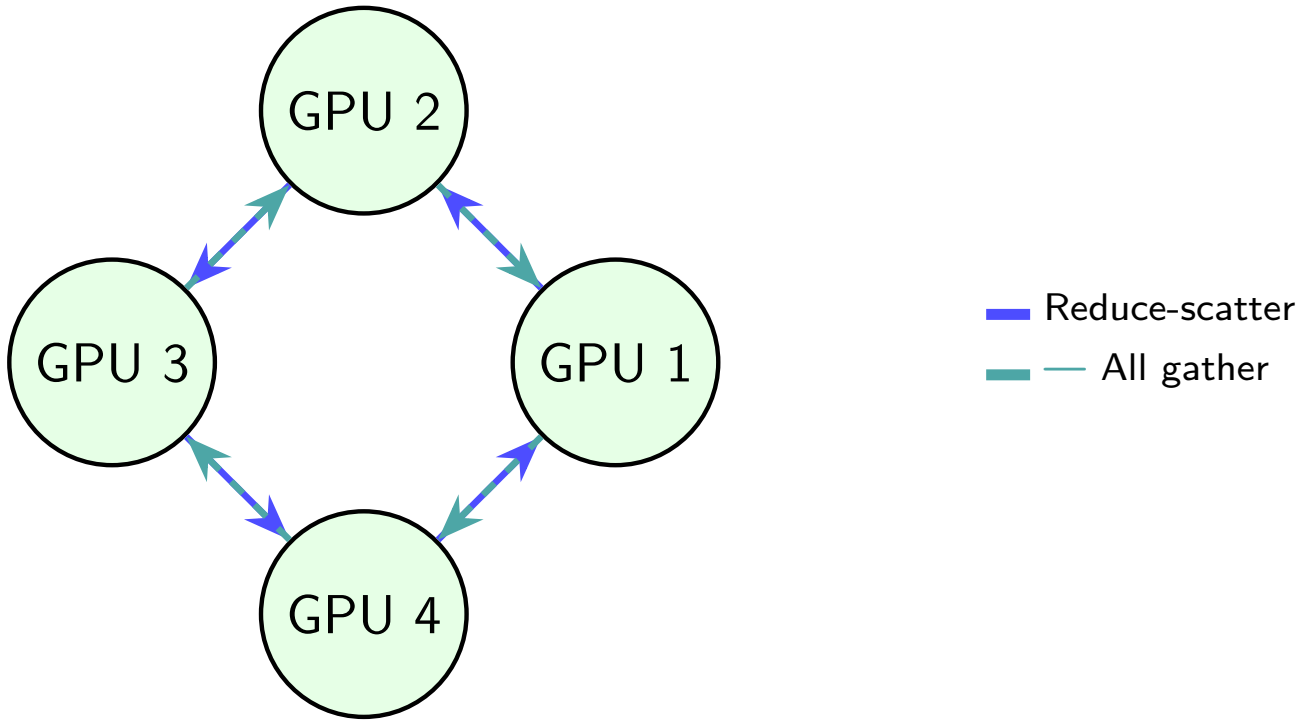
In practice, when synchronizing gradients or parameters across GPUs, we perform the same averaging - but via the **Ring-AllReduce** protocol.

Consensus theory $\Rightarrow$ *Efficient synchronization in deep learning clusters.*

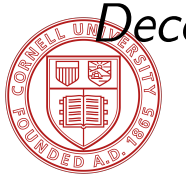


Globally average in practice: Ring-AllReduce

Idea: All GPUs share gradients in a ring to compute the global average. No central server; synchronization is peer-to-peer.



Key takeaway: Each GPU eventually holds $\bar{g} = \frac{1}{n}\sum_i g_i$ and updates $\mathbf{x}$.
Decentralized synchronization (common in datacenter cluster training).



Ring-AllReduce Phase 1: Reduce-scatter

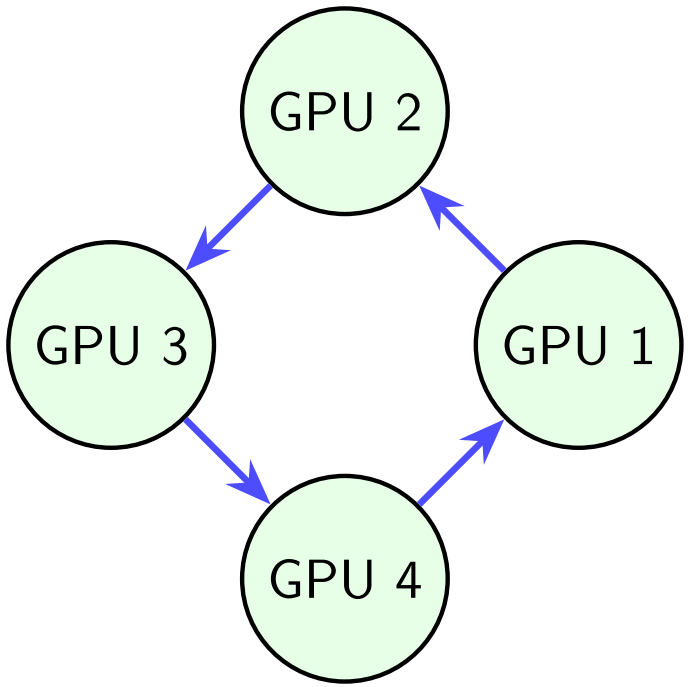
Goal: Compute partial sums of gradient chunks in a distributed way.

- Each GPU splits its gradient into n parts:

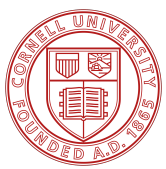
$$g_i = [g_i^{(1)}, g_i^{(2)}, \ldots, g_i^{(n)}].$$

- GPUs pass and sum chunks around ring.
- After $n-1$ steps, each GPU holds one summed chunk $\sum_i g_i^{(k)}$.

Analogy: Everyone edits one page of an essay.



Result: Each GPU stores $\frac{1}{n}$ of the total gradient sum.



Ring-AllReduce Phase 2: All-Gather

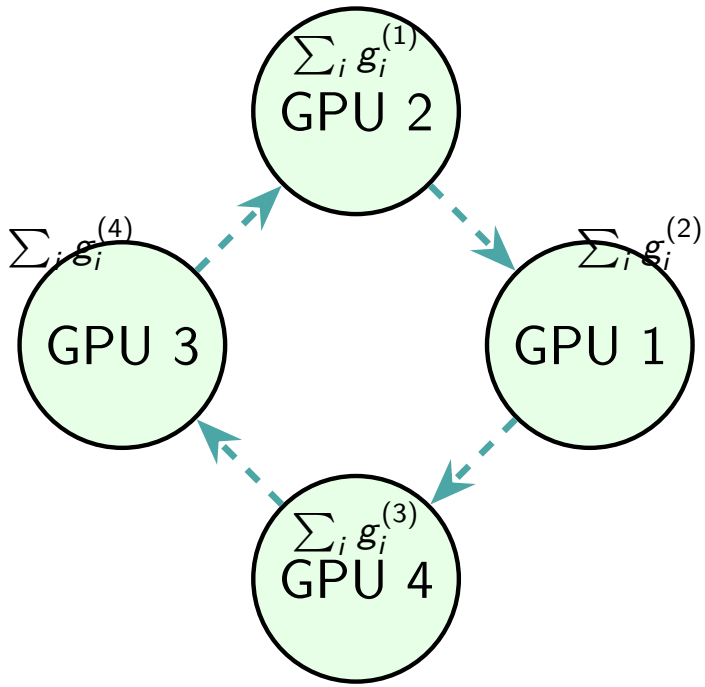
Goal: Share the summed chunks so that all GPUs obtain the full averaged gradient.

- Each GPU holds one chunk $\sum_i g_i^{(k)}$.
- GPUs circulate these chunks around ring.
- After $n-1$ passes, all GPUs have all chunks:

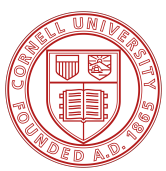
$$[\sum_i g_i^{(1)}, \sum_i g_i^{(2)}, \ldots, \sum_i g_i^{(n)}].$$

- Each divides by n to get $\bar{g} = \frac{1}{n}\sum_i g_i$.

Analogy: Everyone exchanges their completed page to rebuild the full essay.

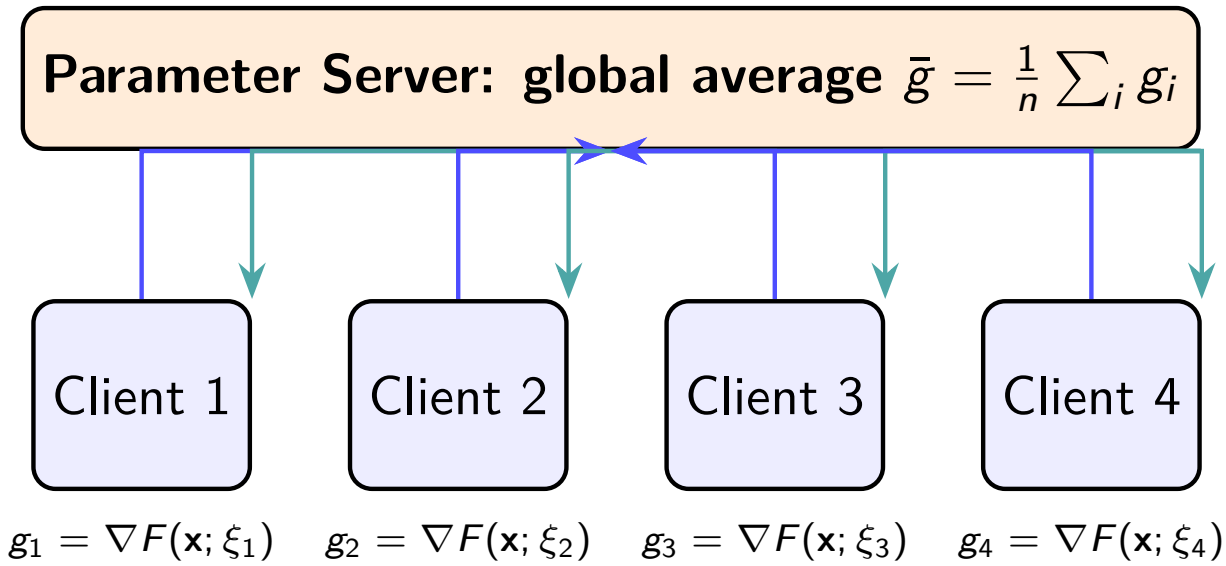


Result: Each GPU has the full averaged gradient $\bar{g}$ and updates $\mathbf{x}$.



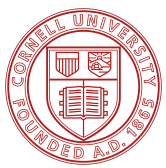
Globally average in practice: Parameter server

Idea: Each client computes local gradients, sends them to a central server, which averages and broadcasts the updated global model.



All clients synchronize per iteration

Key takeaway: A single global node (server) performs the averaging and distributes $\mathbf{x}^{k+1}$. *Common in Federated Learning.*



Parallel SGD convergence rate

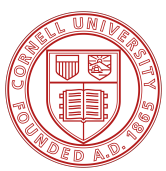
Assumptions:

- F: μ-strongly convex, L-smooth
- $g(\mathbf{x};\xi)$ is an unbiased estimate of $\nabla F(\mathbf{x})$, with bounded variance.

Theorem 1 (Parallel SGD under smooth and convex loss)

Let f be L-smooth and $\eta_k \equiv \eta = 1/\sqrt{K}$. Then the following holds

$$\frac{1}{K}\sum_{k=1}^{K}\mathbb{E}[||\nabla f(\mathbf{x}^k)||_2^2] \leq \mathcal{O}\left(\frac{1}{K} + \frac{\sigma}{\sqrt{nK}}\right)$$



Linear speedup in parallel SGD

Theorem 1 (Parallel SGD under smooth and convex loss)

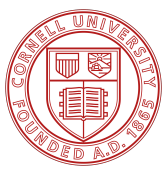
Let f be L-smooth and $\eta_k \equiv \eta = 1/\sqrt{K}$. Then the following holds

$$\frac{1}{K}\sum_{k=1}^{K}\mathbb{E}[||\nabla f(\mathbf{x}^k)||_2^2] \leq \mathcal{O}\left(\frac{1}{K} + \frac{\sigma}{\sqrt{nK}}\right)$$

This implies that to achieve an ϵ-accurate solution, Parallel SGD needs

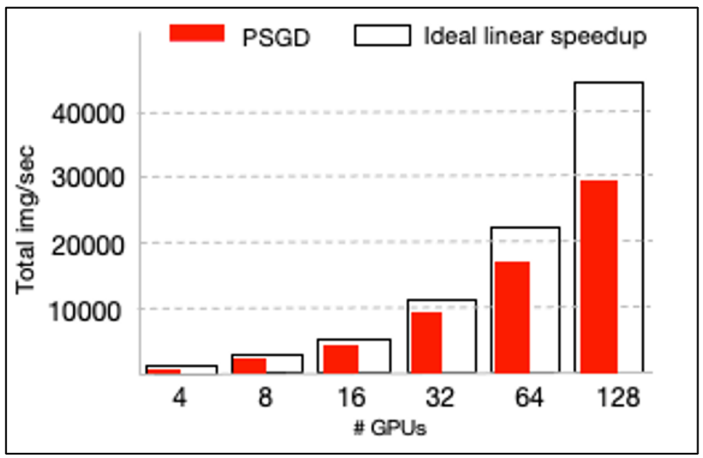
$$\frac{\sigma}{\sqrt{nK}} \leq \epsilon \quad \Longrightarrow \quad K \geq \frac{\sigma^2}{n\epsilon^2} \quad \text{iterations,}$$

which decreases linearly with number of nodes n; called *linear speedup*!

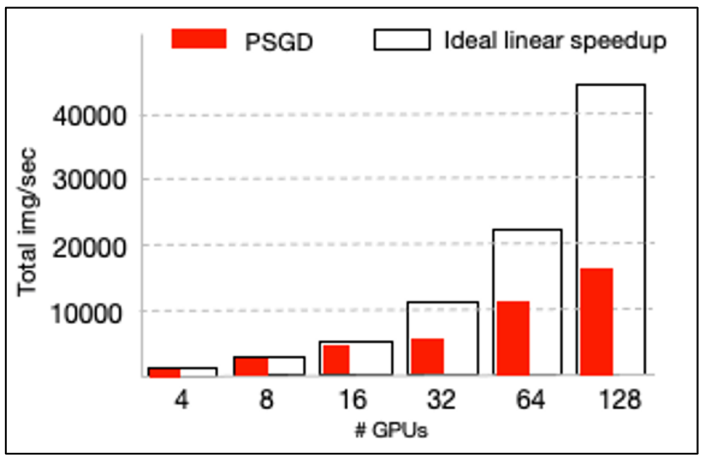


Parallel SGD may not achieve actual linear speedup

- Cannot achieve ideal linear speedup due to communication overhead
- Larger communication-to-computation ratio $\rightarrow$ worse performance

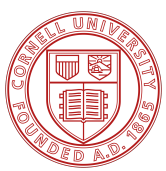


Small comm.-to-compt. ratio



Large comm.-to-compt. ratio

- Ring-all reduce is used in each experiment!
- How can we reduce the communication overhead in Parallel SGD?



Methodologies to save communication

- Global average incurs $O(n)$ comm. overhead; proportional to network size n **[Decentralized communication]**
- Each node interacts with the server at every iteration; proportional to iteration numbers **[Lazy communication]**
- Each node sends a full model (or gradient) to the server; proportional to dimension d **[Compressed communication]**
- and more (asynchronous communication; robust communication against Byzantine nodes, etc.)

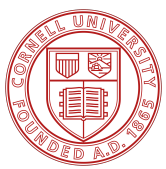
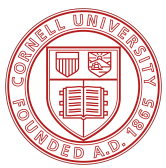


Table of Contents

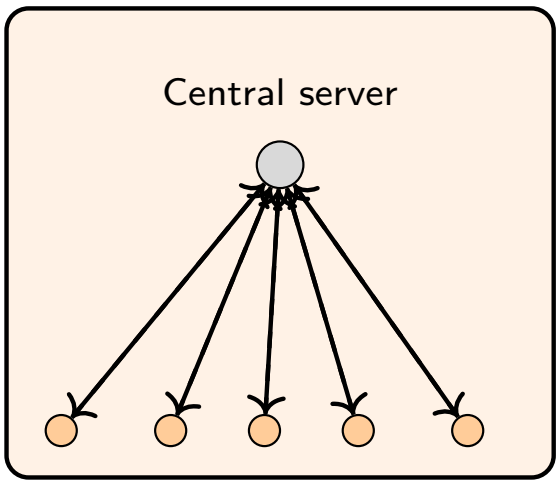


Optimization formulation of data parallelism

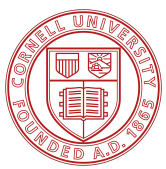
A network of n nodes (such as mobile devices) collaborate to solve:

$$\min_{\mathbf{x}\in\mathbb{R}^d} f(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} f_i(\mathbf{x}), \quad \text{where } \boxed{f_i(\mathbf{x}) = \mathbb{E}_{\xi_i\sim D_i}[F(\mathbf{x};\xi_i)]}$$

- Each component $f_i : \mathbb{R}^d \to \mathbb{R}$ is local and private to node i.
- Random variable ξ_i denotes local data following distribution D_i.
- D_i may be different $\Rightarrow$ **data heterogeneity**.

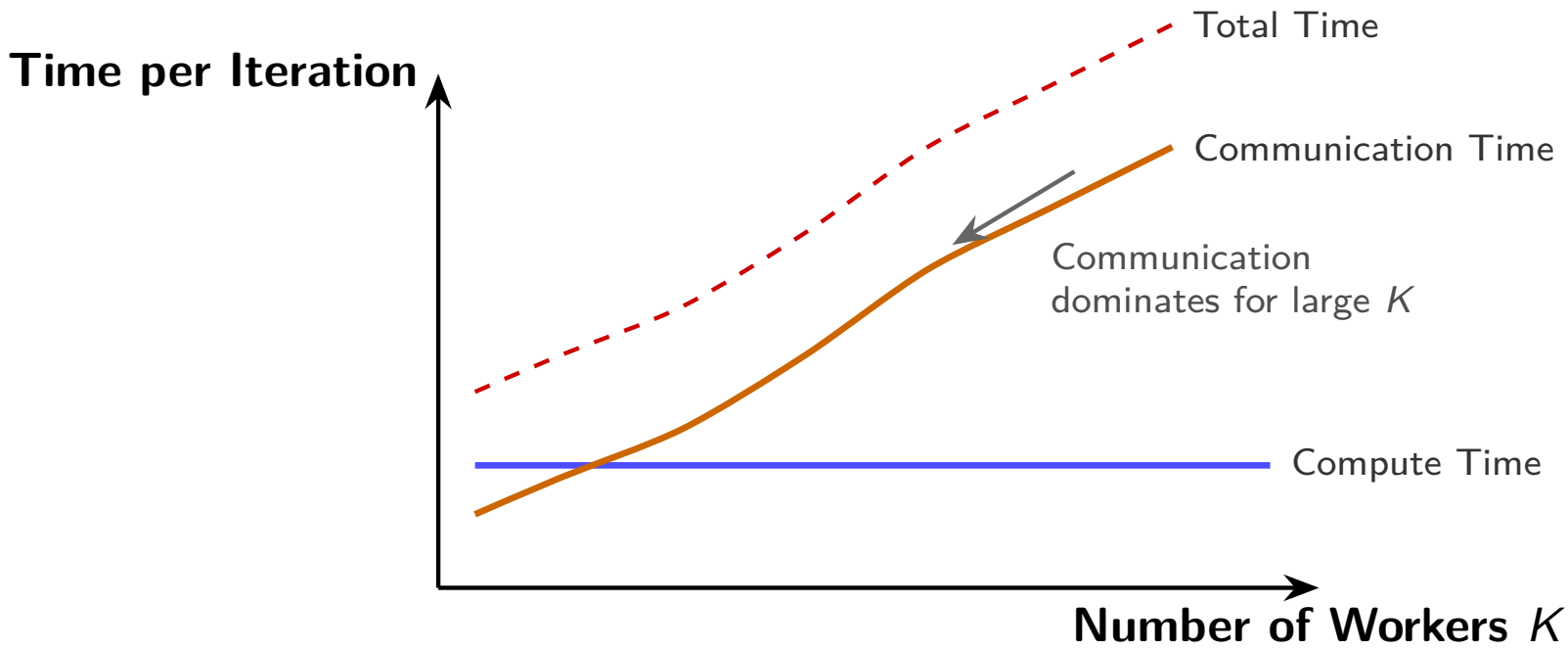


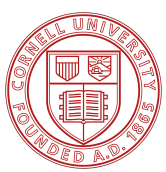
Local data on nodes



Communication bottleneck in Parallel SGD

- In Parallel SGD, workers **synchronize after every step**.
- Comm. dominates runtime when n is large or network is slow.





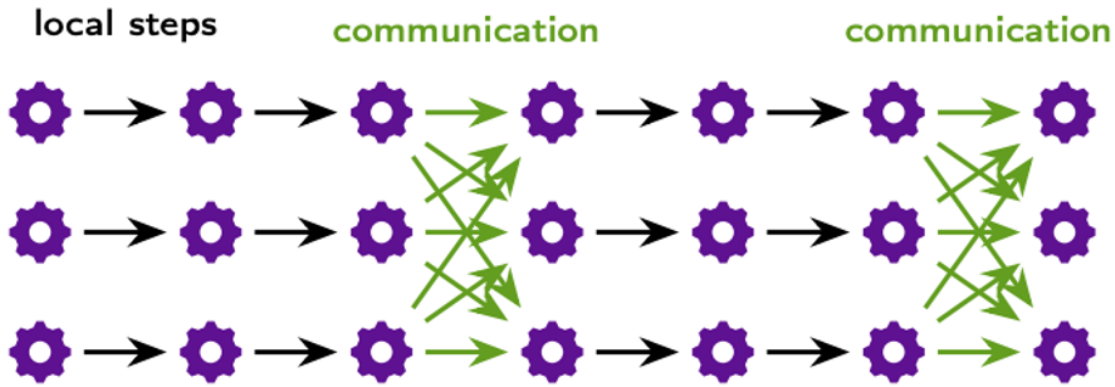
Idea: Local updates before synchronization

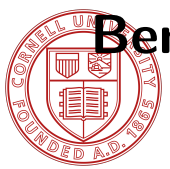
Key idea: Each node i performs several SGD steps before averaging.

$$\mathbf{x}_i^{(s+1)} = \mathbf{x}_i^{(s)} - \eta\, g_i^{(s)}, \quad s = 0, \ldots, \tau - 1$$

where $\mathbf{x}_i^{(0)} = \mathbf{x}^k$. After every τ steps:

$$\mathbf{x}^{k+1} = \frac{1}{n} \sum_{i=1}^{n} \mathbf{x}_i^{(\tau)}$$





Benefit: Reduces communication by a factor of τ.

Mini-batch SGD vs. Local SGD

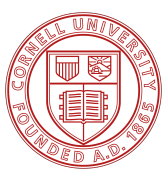
Mini-batch or Parallel SGD:

$$\mathbf{x}^{t+1} = \mathbf{x}^t - \eta_t \frac{1}{n\tau} \sum_{i=1}^{n} \sum_{s=1}^{\tau} \nabla F(\mathbf{x}^t; \xi_i^{t,s})$$

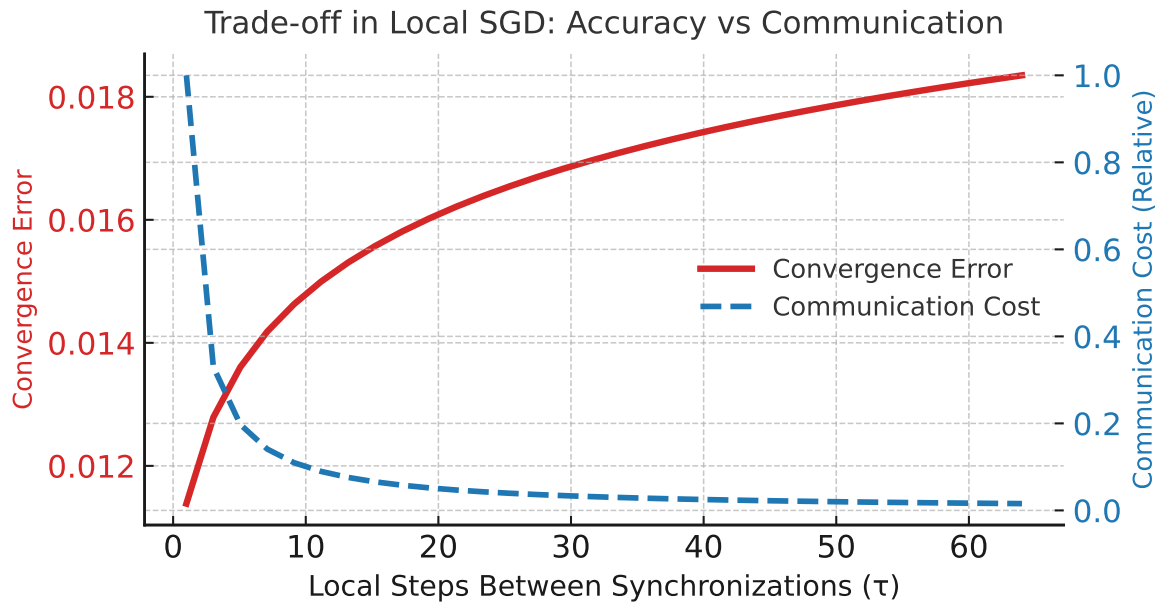
Local SGD:

$$\mathbf{x}_i^{t+1} = \begin{cases} \mathbf{x}_i^t - \eta_t \nabla F(\mathbf{x}_i^t; \xi_i^t), & t \bmod \tau \neq 0 \\ \frac{1}{n} \sum_{i=1}^{n} \left(\mathbf{x}_i^t - \eta_t \nabla F(\mathbf{x}_i^t; \xi_i^t) \right), & t \bmod \tau = 0 \end{cases}$$

Method	Mini-batch SGD	Local SGD
# Comm. rounds	K	K
Batch size	$n\tau$	n
# Model updates	K	τK
# Gradient calcs	$n\tau K$	$n\tau K$

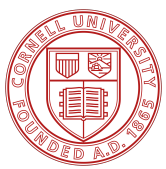


Communication vs. computation trade-off



$$\text{Runtime per iteration} = \text{Compute} + \frac{1}{\tau}\text{Comm.}$$

Insight: Increasing τ improves efficiency but risks model drift.



Why Local SGD works under homogeneous data?

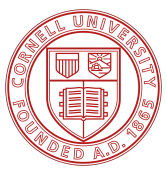
If $f(\mathbf{x})$ is convex and all workers start synchronized ($\mathbf{x}_i^t = \bar{\mathbf{x}}^t$):

$$f(\bar{\mathbf{x}}^{t+\tau}) \leq f(\bar{\mathbf{x}}^t) - (\text{averaged descent term}).$$

Thus, synchronization preserves global descent.

$$f(\bar{\mathbf{x}}^{t+\tau}) \leq \frac{1}{n}\sum_{i=1}^{n} f_i(\mathbf{x}_i^{t+\tau}) \leq f(\bar{\mathbf{x}}^t) - (\text{progress}).$$

Not true in general if f_i differ across workers (non-i.i.d.).



Quadratic objectives: analytical insight

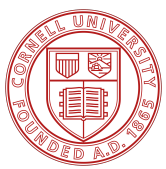
For local quadratic objectives $f_i(\mathbf{x}) = \frac{1}{2}\mathbf{x}^\top A_i \mathbf{x} - \mathbf{b}_i^\top \mathbf{x}$:

$$\mathbf{x}_i^{k+1} = \mathbf{x}_i^k - \eta_k \nabla f_i(\mathbf{x}_i^k).$$

Averaging yields:

$$\mathbb{E}[\bar{\mathbf{x}}^{k+1} | \mathcal{F}^k] = \bar{\mathbf{x}}^k - \eta_k \frac{1}{n} \sum_{i=1}^n \nabla f_i(\mathbf{x}_i^k) \approx \bar{\mathbf{x}}^k - \eta_k \nabla f(\bar{\mathbf{x}}^k).$$

Hence, Local SGD mimics global descent dynamics.



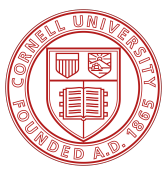
Local SGD improves efficiency in quadratic setting

Theorem 2 (Local SGD under smooth and convex loss)

The error bound for Local SGD with τ local updates equals the bound for Mini-batch SGD with batch size n and $K\tau$ rounds:

$$\epsilon_{\text{L-SGD}} := \frac{1}{K}\sum_{k=1}^{K}\mathbb{E}[||\nabla f(\mathbf{x}^k)||_2^2] = \mathbf{\Theta}\left(\frac{1}{K\tau} + \frac{\sigma}{\sqrt{nK\tau}}\right)$$

- More local updates τ always help convergence.
- Mini-batch SGD: $\epsilon_{\text{MB-SGD}} = \mathbf{\Theta}\left(\frac{1}{K} + \frac{\sigma}{\sqrt{nK\tau}}\right)$
- Therefore, Local SGD *can be better* given the same computation budget.



Performance for general convex objectives*

Upper and lower bounds for Local SGD:

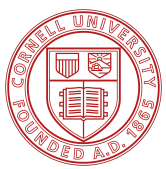
$$\text{Upper:} \qquad \epsilon_{\text{L-SGD}} = \mathcal{O}\left(\frac{\sigma^{2/3}}{K^{2/3}\tau^{1/3}} + \frac{\sigma}{\sqrt{nK\tau}}\right)$$

$$\text{Lower:} \qquad \Omega\left(\frac{\sigma^{2/3}}{K^{2/3}\tau^{2/3}} + \frac{\sigma}{\sqrt{nK\tau}}\right)$$

Mini-batch SGD: $\quad \boldsymbol{\Theta}\left(\frac{1}{K} + \frac{\sigma}{\sqrt{nK\tau}}\right)$

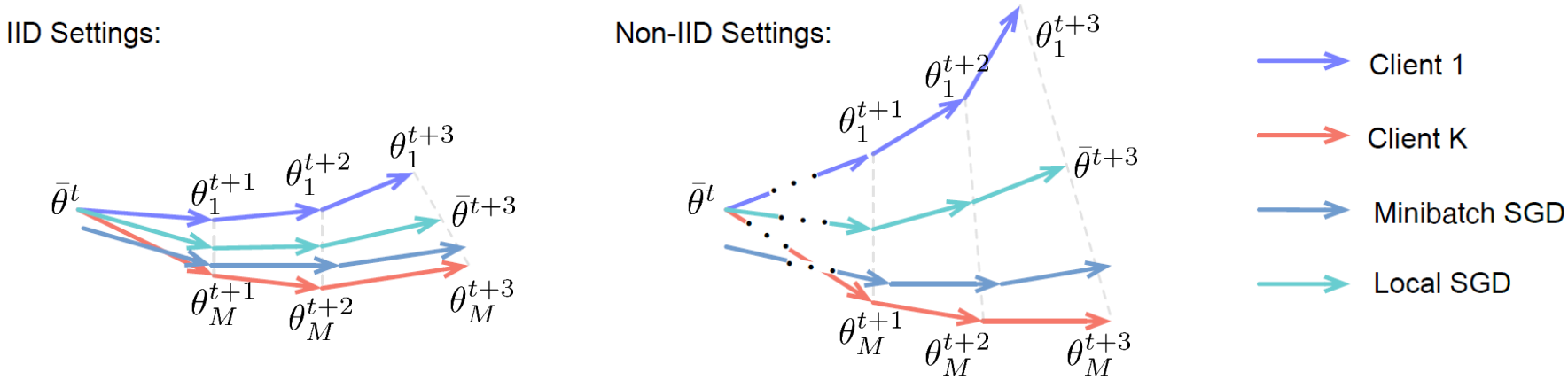
Local SGD better when $K \lesssim \tau$, worse when $K \gtrsim \tau$.

Woodworth et al. "Is Local SGD Better than Mini-batch SGD?", *ICML 2020*

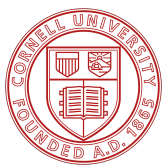


Why local SGD may fail under heterogeneous data?

In heterogeneous (non-i.i.d.) data settings, local gradients are misaligned

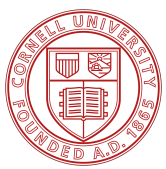


- Local updates diverge due to heterogeneous data (Γ^2).
- Need additional assumptions to control gradient dissimilarity.
- Larger $\tau \Rightarrow$ greater deviation from the global model.

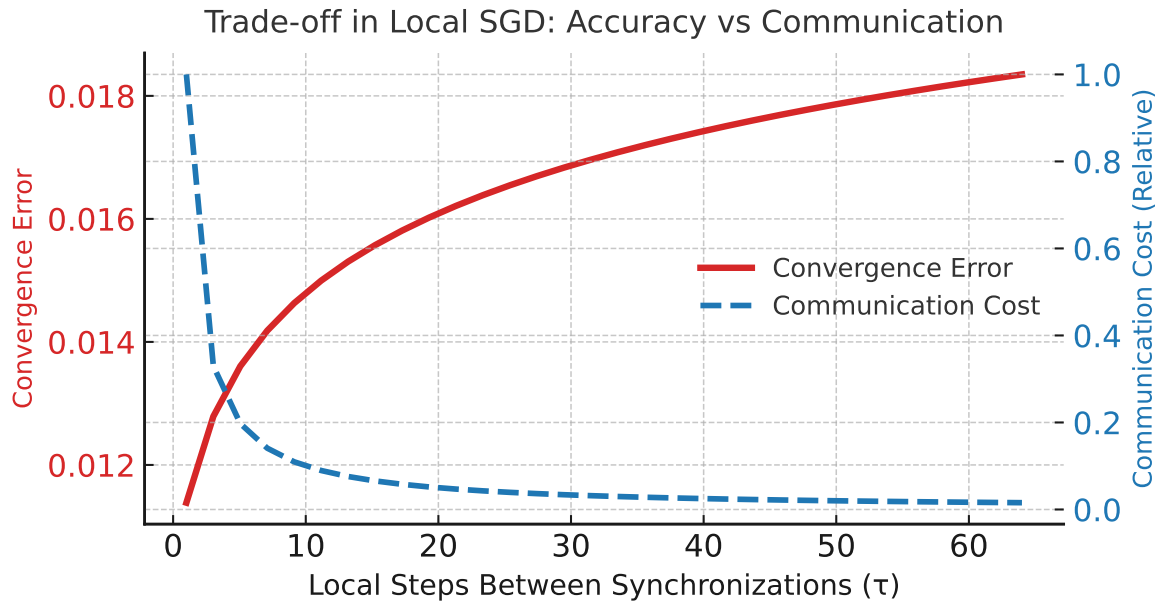


Summary: Parallel SGD vs local SGD

Aspect	**Parallel SGD**	**Local SGD**
Communication	Every iteration	Every τ iterations
Local computation	1 gradient step	τ local steps
Speed	Communication-limited	Compute-efficient
Convergence rate	Stable	Slower ($(\eta^2\tau\Gamma^2)$ bias)
Best for	Data centers, i.i.d. data	Federated settings

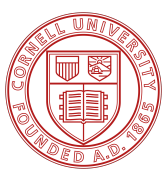


Takeaway: Communication - accuracy trade-off



- $\tau = 1$: fully synchronized (Parallel SGD)
- $\tau > 1$: fewer syncs $\Rightarrow$ faster but drift grows
- Choose τ based on network bandwidth and data heterogeneity

Rule of thumb: $\tau^* \propto \sqrt{\frac{c_{\text{comm}}}{c_{\text{comp}}}}$



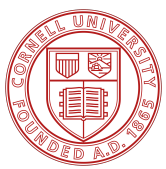
When to use Local SGD?

Recommended if:

- Communication cost $c_{\text{comm}} \gg c_{\text{comp}}$
- Data across workers are relatively homogeneous
- Occasional synchronization suffices for convergence

Avoid if:

- Highly non-i.i.d. data (strong gradient heterogeneity)
- Models are unstable to small parameter changes



Recap and fine-tuning

- What we have talked about today?

  $\Rightarrow$ **Data parallelism** enables scaling model training across multiple workers by distributing data and synchronizing models through averaging.

  $\Rightarrow$ **Parallel SGD** achieves global model updates via synchronization.

  $\Rightarrow$ **Local SGD** relaxes synchronization - maintaining learning efficiency in homogeneous data; drift apart in heterogeneous settings.

Welcome anonymous survey!